Tutorial 5

Sorting algorithms: trade-offs, edge cases, and micro-benchmarks

18YZALG – Basics of Algorithmization, Summer Semester 2026

How today works

- We will compare five classic sorting algorithms.
- For each: **idea** → **properties** → **edge cases** → **measured examples**.
- Goal: learn to answer *"which sort should I use here?"* with reasons.

Ground rule

We care about **scaling and trade-offs**. One timing number is never the full story.

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1. Sorting problem + vocabulary (stable, in-place, adaptive).
2. Micro-benchmarking rules (how to measure fairly).
3. QuickSort and MergeSort (divide & conquer).
4. Insertion / Selection / Bubble (quadratic family, but with useful niches).
5. Summary: a practical decision table.

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Why sorting shows up everywhere

- Preprocessing step: **sort once, query many times**. After sorting, operations like binary search, range queries, and grouping become straightforward and fast.
- Data work: ranking top-k items, deduplicating records, computing medians/percentiles, and performing merge-style joins all naturally rely on sorted order.
- Engineering benefit: a sorted sequence reveals patterns (runs, gaps, outliers), so debugging and validating intermediate results is often easier.
- Design benefit: many algorithms become simpler and more reliable when they can assume monotonic order in the input.

One-liner

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The sorting task (spec)

Specification

```
Input: list A of n comparable items
Output: permutation B of A such that
        B[0] <= B[1] <= ... <= B[n-1]

Optional constraints:
  - Stable? (keep relative order of equal keys)
  - In-place? (allowed extra memory?)
  - Adaptive? (gets faster on nearly-sorted inputs)
```

Vocabulary you will use to justify a choice

Stable

If $A[i]$ and $A[j]$ compare equal and $i < j$, then they appear in the same order in the output.

In-place

Uses $\mathcal{O}(1)$ or $\mathcal{O}(\log n)$ extra memory (beyond input).

Adaptive

Runs much faster if data is *already* (almost) sorted.

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Benchmark focus for this tutorial

- We assume the standard measurement protocol from earlier tutorials.
- Today, the key variable is **input shape**: random, nearly sorted, already sorted, many equal keys.
- We compare **growth trends** and cross-over points, not one headline timing.
- For QuickSort, pivot policy is part of the algorithm and must be reported explicitly.

Interpretation lens

The goal is to explain **why** an algorithm wins on a dataset, not just to rank implementations.

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A minimal timing harness (Python)

Code

```
import time, gc

def time_one(sort_func, data, repeats=7):
    gc.disable()
    try:
        best = float('inf')
        for _ in range(repeats):
            A = data[:]              # same input
            t0 = time.perf_counter()
            sort_func(A)
            best = min(best, time.perf_counter() - t0)
        return best
    finally:
        gc.enable()
```

A useful mental model

- Comparison sorting has a hard lower bound: $\Omega(n \log n)$ comparisons.
- So $n \log n$ behavior (QuickSort/MergeSort family) is the right baseline for large, general inputs.
- Scale intuition: at $n = 10{,}000$, $n \log_2 n \approx 133{,}000$, while $n^2 = 100{,}000{,}000$.
- Constants and input shape still matter.

Decision lens

Use asymptotics first, then refine by data shape and constraints (stability, memory, worst-case risk).

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QuickSort (idea)

Divide & Conquer: split into smaller similar subproblems, solve recursively, then combine.

Divide & conquer via partition

Pick a **pivot**. Rearrange the array A so:

$$\{x \in A \mid x < p\} \;\mid\; p \;\mid\; \{x \in A \mid x \geq p\}$$

Then recursively sort the two sides.

- Average case is excellent, but the pivot choice matters.

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QuickSort (pseudocode)

Pseudocode (random pivot)

```
QUICKSORT(A, lo, hi):          # sort A[lo:hi)
  if hi - lo <= 1: return
  p = random index in [lo, hi)
  pivot = A[p]
  i = PARTITION(A, lo, hi, pivot)
  QUICKSORT(A, lo, i)
  QUICKSORT(A, i+1, hi)
```

Invariant (partition)

After partition: all indices $< i$ hold values $<$ pivot, and all indices $> i$ hold values $\geq$ pivot.

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QuickSort (properties)

- Average time: $\mathcal{O}(n \log n)$
- Worst time: $\mathcal{O}(n^2)$ (bad pivots)
- Extra memory: recursion stack $\mathcal{O}(\log n)$ average
- Not stable (standard in-place partition)

When it shines

Fast in practice on random-ish data; in-place; good locality.

When it hurts

Adversarial order + unlucky pivot strategy; many equal keys without 3-way partition.

Measured example: random input (all 5 algorithms)

Random integers (best-of-7; pure-Python reference implementations)

n	Quick	Merge	Insertion	Selection	Bubble
400	**0.280**	0.336	1.34	1.66	3.46
800	**0.615**	0.701	6.04	6.99	15.8
1600	**1.34**	1.60	26.3	28.7	69.3
3200	**2.79**	3.52	108	115	295
4000	**3.72**	4.63	177	187	486

Times are **milliseconds**. Interpretation: on random data, QuickSort / MergeSort scale smoothly, while quadratic sorts explode.

Edge case: "bad pivot" QuickSort

Already sorted input: pivot = first element (a common mistake)

n	Quick (first pivot)	Quick (random)
200	0.787	0.116
400	3.02	0.266
800	9.21	0.588

Takeaway

Pivot strategy is part of the algorithm. The same "QuickSort" can be great or disastrous.

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QuickSort: practical fixes

- **Randomized pivot:** pick a random index in each subarray to avoid predictable worst-case behavior on sorted or adversarial input.
- **Median-of-three pivot:** use the median of first/middle/last values to reduce highly unbalanced splits on partially ordered data.
- **3-way partitioning:** split into <, =, and > regions so equal keys are handled in one pass (fewer useless recursive calls).
- **Depth guard:** if recursion depth becomes too large, fall back to HeapSort to guarantee $\mathcal{O}(n \log n)$ worst-case time.

MergeSort (idea)

Divide & conquer via merging

Split the list in half, recursively sort both halves, then **merge** two sorted lists into one.

- The merge step is linear and very predictable.
- Standard MergeSort is stable.

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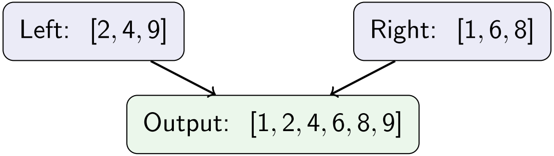
Merge step (picture)



Always take the smaller front element.

MergeSort (pseudocode)

Pseudocode

```
MERGESORT(A):
  if len(A) <= 1: return A
  mid = len(A)//2
  L = MERGESORT(A[:mid])
  R = MERGESORT(A[mid:])
  return MERGE(L, R)

MERGE(L, R):
  i=j=0; out=[]
  while i<len(L) and j<len(R):
    if L[i] <= R[j]: out.append(L[i]); i+=1
    else:            out.append(R[j]); j+=1
  append remaining tail
  return out
```

MergeSort (properties)

- Time: $\mathcal{O}(n \log n)$ in **best/average/worst**
- Extra memory: typically $\mathcal{O}(n)$
- Stable (easy to keep)

Where it wins

Guaranteed worst-case; stable multi-key sorting; great for linked lists; foundation for external sorting.

Where it loses

Needs extra memory; constant factors can be higher than QuickSort.

Stability matters (tiny example)

Scenario: sort by `grade`, keep original order within equal grades

Input (arrival order):

```
[(Alice, 90), (Bob, 90), (Chad, 75), (Dana, 90)]
```

A **stable** sort by grade outputs:

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[(Chad, 75), (Alice, 90), (Bob, 90), (Dana, 90)]
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If you plan to do **multi-key sorting** (sort by secondary key, then primary), stability is essential.

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Insertion Sort (idea)

Grow a sorted prefix

Maintain that $A[0 : i)$ is sorted. Insert $A[i]$ into the right position by shifting bigger items.

- **Stable** if implemented with strict $>$: equal keys are not swapped past each other, so original relative order is preserved.
- **In-place** with $\mathcal{O}(1)$ extra memory: it only shifts values inside the same array and keeps implementation overhead low.
- Adaptive: running time is linked to the number of inversions, so nearly sorted inputs are close to linear in practice.

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Insertion Sort (pseudocode)

Pseudocode

```
INSERTION-SORT(A):
  for i in 1..n-1:
    x = A[i]
    j = i-1
    while j>=0 and A[j] > x:
      A[j+1] = A[j]      # shift right
      j -= 1
    A[j+1] = x
```

Insertion Sort (properties)

- Best-case (already sorted): $\mathcal{O}(n)$
- Worst-case (reverse sorted): $\mathcal{O}(n^2)$
- Extra memory: $\mathcal{O}(1)$
- Often used as a **base case** in hybrid sorts.

When it wins

Small n or nearly sorted data (few inversions).

When it loses

Large random data: quadratic behavior dominates.

Measured example: nearly sorted input (1% swaps)

Nearly sorted integers (best-of-7; ms)

n	Quick	Merge	Insertion	Selection	Bubble
400	0.277	0.260	**0.0490**	1.77	1.60
800	0.589	0.571	**0.241**	7.39	8.57
1600	1.35	1.27	**0.610**	29.1	32.8
3200	2.80	2.76	2.68	121	174
6400	5.94	6.04	13.4	483	733

Interpretation: InsertionSort is great when "almost sorted" really means "few inversions".

Selection Sort (idea)

Repeatedly pick the minimum

For position i, find the minimum of $A[i : n)$ and swap it into $A[i]$.

- Always does $\Theta(n^2)$ comparisons (input order does not help).
- In-place with only $\mathcal{O}(n)$ swaps.
- Usually **not stable** (swap breaks equal-key order).

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Selection Sort: when could you justify it?

- When **writes are expensive** (e.g., swapping large records on slow storage).
- When you need a very small, predictable piece of code.
- Educational value: makes the n^2 cost painfully clear.

Otherwise

If writes are cheap (typical RAM), there is almost always a better choice.

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Measured example: selection minimizes swaps (writes)

Random integers: number of data movements (not time)

n	Selection swaps	Bubble swaps	Insertion shifts
100	94	2220	2220
200	198	10536	10536
400	392	41772	41772
800	791	161039	161039

Interpretation

If **writes are expensive** (e.g., swapping huge records on slow storage), SelectionSort's $\Theta(n)$ swaps can beat methods that do $\Theta(n^2)$ data movements.

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Bubble Sort (idea)

Swap adjacent inversions

Repeatedly scan the array and swap any adjacent out-of-order pair.

- Stable (only swaps adjacent items).
- Quadratic worst-case: $\mathcal{O}(n^2)$.
- With an "early exit" flag, best-case is $\mathcal{O}(n)$.

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Bubble Sort (early exit variant)

Pseudocode

```
BUBBLE-SORT(A):
  for pass in 1..n:
    swapped = False
    for j in 0..n-pass-1:
      if A[j] > A[j+1]:
        swap(A[j], A[j+1])
        swapped = True
    if not swapped:
      return   # already sorted
```

Edge case: already sorted input

Already sorted integers (best-of-7; ms)

n	Bubble (early)	Insertion	Merge	Quick
800	**0.0255**	0.0445	0.517	0.593
1600	**0.0545**	0.0904	1.12	1.35
3200	**0.110**	0.187	2.35	2.67
6400	**0.224**	0.395	4.84	5.79
12800	**0.446**	0.765	10.6	13.6

Interpretation

BubbleSort becomes a linear-time **sortedness check**. But on real unsorted data it is still a poor general sort.

Edge case: already sorted input

Already sorted integers (best-of-7; ms)

n	Bubble (early)	Insertion	Merge	Quick
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1600	**0.0545**	0.0904	1.12	1.35
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Bubble Sort: where it does not belong

- General-purpose sorting of medium/large arrays.
- Any situation with a meaningful time limit.
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Comparison table (properties)

Algorithm	Stable	In-place	Adaptive	Worst-case time
QuickSort (std)	no	yes	no	$\mathcal{O}(n^2)$
MergeSort	yes	no (extra n)	no	$\mathcal{O}(n \log n)$
Insertion	yes	yes	**yes**	$\mathcal{O}(n^2)$
Selection	no (typically)	yes	no	$\mathcal{O}(n^2)$
Bubble (early)	yes	yes	some (best-case)	$\mathcal{O}(n^2)$

Choosing in practice (rules of thumb)

- Need stability (multi-key, preserve order) $\Rightarrow$ MergeSort-family.
- Need in-place and great average speed $\Rightarrow$ QuickSort with good pivots.
- Data is almost sorted or n is tiny $\Rightarrow$ InsertionSort.
- Writes are expensive $\Rightarrow$ SelectionSort can be justified (rare).
- Already sorted detection $\Rightarrow$ Bubble early-exit (or just one linear pass check).

Meta-rule

Pick based on **constraints**: size, memory, stability, data "shape", and worst-case risk.

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Pick based on **constraints**: size, memory, stability, data "shape", and worst-case risk.

Quick check (predict the winner)

For each scenario, name a likely good choice (and why)

1. $n = 200$, almost sorted, only a few swaps.
2. $n = 50000$, random order, memory is fine.
3. You sort records by (`last_name, first_name`) using two passes.
4. You suspect inputs may be adversarial (someone tries to slow you down).

Likely answers: 1) InsertionSort; 2) QuickSort (with good/random pivot) or MergeSort; 3) MergeSort; 4) MergeSort.

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Wrap-up

- "Same problem" does not mean "same best algorithm".
- Benchmarks should support a story about **scaling**.
- Next: **Heap** data structure and HeapSort (worst-case guarantee; priority-queue power).