Exam instructions

This document contains the complete list of exam questions for **01LIP – Linear Programming**. Practical problems are marked with the symbol 🔨. Their specific statements will be provided in the exam paper.

The exam consists of **two theoretical questions and one practical problem**. Each lettered item below is one question. The topic groups are provided for orientation only. Theorem, lemma, and proposition numbers refer to the English edition of the 01LIP textbook.

1. **Formulating a linear program**
   - a) Write a general LP maximization problem in inequality form and equality form. Define its feasible set and an optimal solution. Explain how to convert between the two forms.
   - b) 🔨 Convert the given problem to the specified maximization inequality or equality form with nonnegative variables.
2. **The geometry of linear programming**
   - a) For $P = \{\mathbf{x} \geq \mathbf{0} : A\mathbf{x} = \mathbf{b}\}$, where $A \in \mathbb{R}^{m \times n}$ has rank m, define a basis, a basic feasible solution (BFS), and a degenerate BFS. State and prove Theorem 2.16 on the uniqueness of the basic solution determined by a given basis.
   - b) State and prove the fundamental theorem of linear programming (Theorem 3.1).
   - c) State and prove Theorem 3.2 on the equivalence between vertices of the feasible polyhedron and basic feasible solutions.
   - d) 🔨 Solve the given LP in two variables graphically. Find all optimal solutions and justify their optimality, or justify infeasibility or unboundedness.
3. **The simplex method**
   - a) Define a simplex tableau and explain its components (Lemma 4.5). For a feasible basis, state criteria for optimality and unboundedness. Describe the ratio test (Lemma 4.6), the meaning of a degenerate step, and the purpose of an anti-cycling rule.
   - b) Describe the construction of an auxiliary problem with artificial variables and the two-phase simplex method. State and prove the criterion for feasibility of the original problem in terms of the optimal value of Phase I (Proposition 5.1). Explain the big-M method, the lexicographic meaning of the symbol M, and the relationship between the two methods.
   - c) 🔨 Using the given simplex tableau and the specified phase of the computation, determine whether the original problem is infeasible, unbounded, or has a finite optimum. Find an optimal solution, or justify infeasibility or unboundedness.
   - d) 🔨 For the given LP, formulate the corresponding big-M auxiliary problem and find an initial feasible basis for the augmented problem. If required by the problem statement, solve Phase I and determine whether the original problem is feasible.

4. **Linear programming duality**

   a) Formulate a general LP maximization problem in standard inequality form and derive its dual. State the weak duality theorem (Theorem 7.3) and the strong duality theorem (Theorem 7.4). Prove the weak duality theorem.

   b) For a standard primal–dual pair, state the complementary slackness conditions (Theorem 7.5). Prove that a pair of feasible solutions is optimal if and only if these conditions hold. Explain how to use them to verify optimality.

   c) Write the dual of the given primal LP, including the correct inequality directions and variable domains. If a primal solution and a dual solution are given, check their feasibility and determine whether they are optimal.

5. **The transportation problem**

   a) Formulate the balanced transportation problem as an LP: define the decision variables, objective function, and constraints. Briefly describe the main properties of the transportation problem.

   b) Construct the constraint matrix and a structured transportation tableau for the given transportation problem.

6. **Zero-sum matrix games and the minimax theorem**

   a) Define a zero-sum matrix game and the necessary concepts. Write the LPs for both players and explain the relationship between the strong duality theorem (Theorem 7.4) and the minimax theorem (Theorem 10.11).

   b) For the given payoff matrix, formulate the LPs for both players and find or verify optimal strategies and the value of the game. Follow the payoff convention specified in the problem statement.

7. **Integer and mixed-integer linear programming**

   a) Formulate an integer linear program (ILP) and a mixed-integer linear program (MILP), and describe their feasible sets. Define the LP relaxation and explain the bound it provides on the optimal value. State when an optimal solution of the relaxation also solves the original problem.

   b) Describe the principle and basic scheme of the branch-and-bound method.

   c) Describe the principle and basic scheme of the Gomory cutting-plane method.

   d) For the given ILP or MILP, solve the LP relaxation and carry out the specified branching. State the bounds at the nodes and justify pruning them. If a complete solution is required, prove optimality of the feasible solution found that satisfies the required integrality conditions, or prove infeasibility.